Trisect a Line Segment (1)

Method:

1) Let AB be a line segment
2) Find Midpoint of segment AB, let's call it C
3) Construct a circle with center B and radius BC, Label the intersection of circle B and line AB as point D
4) Construct a circle with center A and radius AB, then another circle with center D and radius AD. Let's label the intersections of circle A and circle D as E and F
5) Construct a line segment EF, label the intersection of segment AB and segment EF as G
6) Construct a circle with center G and radius AG, label the intersection of circle G and segment AB as H.

Claim: Points G and H trisects segment AB.

Proof:

We want to show that AB = 3AG
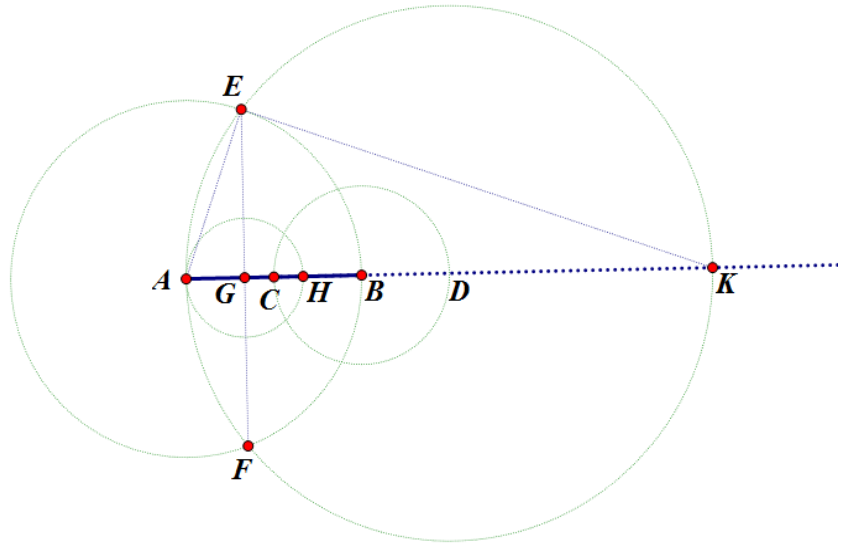
Since C is the midpoint of AB,

$$AC = BC$$

BC and BD are the radii of the same circle, so

$$BD = BC = AC$$

Furthermore, $\quad AD = 3AC$.

Since segment KE is tangent to circle A of radius AE, we know that AE⊥KE. Hence ΔAKE is a right triangle. Additionally, GE⊥AB, hence GE⊥AK. Thus we have similar triangles ΔAKE and ΔAGE. So for similar triangles we get:

$$\frac{AG}{AE} = \frac{AE}{AK}$$

$$\Longrightarrow \frac{AG}{AB} = \frac{AB}{6AC}$$

$$\Longrightarrow \frac{AG}{AB} = \frac{2AC}{6AC}$$

Hence, $AB = 3AG$